Quiz 8 (10pts)
Math 214 Section Q1 Winter 2010

Your name:_____ ID#:_____

Please, use the reverse side if needed.

1.(5 pts) Find the length of the curve $\mathbf{r}(t) = \cos t^2\mathbf{i} + \sin t^2\mathbf{j} + t^2\mathbf{k}$, $0 \le t \le 1$.

Solution.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = -2t\sin t^2\mathbf{i} + 2t\cos t^2\mathbf{j} + 2t\mathbf{k}$$
$$|\mathbf{v}| = \sqrt{(-2t\sin t^2)^2 + (2t\cos t^2)^2 + (2t)^2}$$
$$= \sqrt{4t^2\sin^2 t^2 + 4t^2\cos^2 t^2 + 4t^2}$$
$$= \sqrt{4t^2 + 4t^2} = 2\sqrt{2}\ t$$

$$L = \int_0^1 |\mathbf{v}|dt = 2\sqrt{2}\int_0^1 tdt = \sqrt{2}\ t^2\big|_0^1 = \sqrt{2}.$$

2.(5 pts) Find the curvature of the curve $\mathbf{r}(t) = \cos t^2\mathbf{i} + \sin t^2\mathbf{j} + t^2\mathbf{k}$, $t > 0$.

Solution.

From above:

$$\mathbf{T} = \frac{1}{|\mathbf{v}|}\mathbf{v} = -\frac{1}{\sqrt{2}}\sin t^2\mathbf{i} + \frac{1}{\sqrt{2}}\cos t^2\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k}$$

$$\frac{d\mathbf{T}}{dt} = -\sqrt{2}\ t\cos t^2\mathbf{i} - \sqrt{2}\ t\sin t^2\mathbf{j}$$

$$\left|\frac{d\mathbf{T}}{dt}\right| = \sqrt{2t^2\cos^2 t^2 + 2t^2\sin^2 t^2} = \sqrt{2}\ t$$

$$\kappa = \frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right| = \frac{\sqrt{2}\ t}{2\sqrt{2}\ t} = \frac{1}{2}.$$